TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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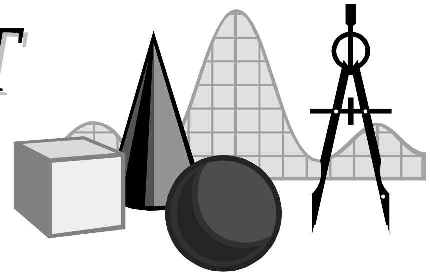

The National Council of Teachers of Mathematics document *Curriculum and Evaluation Standards for School Mathematics* suggests decreased emphasis on factoring of polynomials. However, this does not mean that we should abandon all teaching of factoring. Factoring is a necessary skill and can lead to a deeper understanding of higher-level mathematics. And in spite of the advent of graphing calculators and affordable mathematical software, factoring is still useful. In this issue of *Take It to the MAT*, we will look at a couple of factoring techniques and the connection between them.

When students learn to factor, the first thing drilled into their heads is to always look for a common factor, or more appropriately the *greatest monomial factor* (*GMF*) and the application of the distributive property. After practicing dozens upon dozens of exercises, they know instinctively that $3x^2y$ needs to be factored out of each term of $6x^3y + 18x^3y^3 - 9x^2y$. Even after the introduction of other techniques, such as factoring the difference of squares or perfect trinomial squares, pulling out that common factor can simplify the process. For example, $4x^2 + 24x + 36$ is a perfect trinomial square equal to $(2x+6)^2$, but this is not as easily recognized as is the trinomial after factoring out the GMF, $4\left(x^2 + 6x + 9\right)$.

The concept of the GMF is also helpful in other factoring techniques. In most cases, factoring by grouping is simply applying the distributive property multiple times—often once more than we realize.

Before we get into the details, factor this polynomial: $xz + 2z$. That's simple, see *Figure 1*. We factor out the GMF of *z*, producing the binomial factor $(x+2)$. Now, what about a more complex problem?

$$xz + 2z$$
$$z(x+2)$$

Figure 1

Let's factor the polynomial $xy + 3x + 2y + 6$ (See *Figure 2*). The first two terms are grouped, as are the second two terms. The first group has a GMF of *x*; the second has a GMF of 2. But, there's another common factor! Looking closely, each of the two terms in the last step of *Figure 2* has a common factor of $y + 3$. Basically, is $x(y+3) + 2(y+3)$ really any different than $xz + 2z$ from *Figure 1* above? If we compare the two expressions, there's no reason we couldn't say $z = y + 3$.

$$xy + 3x + 2y + 6$$
$$(xy + 3x) + (2y + 6)$$
$$x(y+3) + 2(y+3)$$

Figure 2

Finally, we can factor out the common factor of $y + 3$ giving the result shown in *Figure 3*. (Some texts will show the final line with the factors reversed as $(x+2)(y+3)$, but it does not connect as well to the factoring of $xz + 2z$ as shown here.)

$$x(y+3) + 2(y+3)$$
$$(y+3)(x+2)$$

Figure 3

Strong connections can be made between factoring by grouping and common factors. This is particularly true of that last step where students frequently stumble.