TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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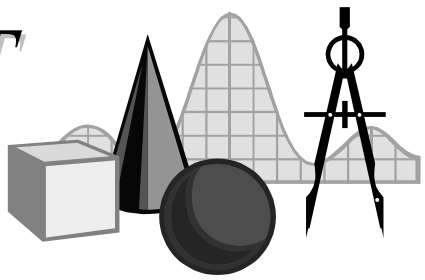

In written language we use words to represent objects, actions, and characteristics of objects and actions. In mathematics we do the same. But just as we can misspell words and baffle the reader, we can do the same in mathematics. The mistake may be subtle. In replacing *their* with *they're* or *there,* the reader may understand what the writer meant, but it may take a couple of readings to make the connection. Similarly, subtle mistakes in mathematical notation may also create confusion or misconceptions.

One of the most common notation mistakes teachers and students make is using equals signs when approximation signs are required. The examples below are frequently seen:

(1) $\frac{2}{3} = 0.666 = 0.67$

(2) $\frac{63}{250} = 25\%$

(3) $\pi = 3.14$, $\pi = 3.1416$, or $\pi = \frac{22}{7}$

All three of these cases are notationally incorrect. In example 1, two-thirds is a repeating decimal—$\frac{2}{3} = 0.666\ldots = 0.\overline{6}$. If one *truncates* the decimal as in 0.666, or *rounds* it as in 0.67, one is giving an *approximation*. Two-thirds **does not *equal*** 0.666 or 0.67. Therefore, one must use an approximation symbol and write either $\frac{2}{3} \approx 0.666$ or $\frac{2}{3} \approx 0.67$. That is, two-thirds *is approximately* 666 thousandths or *is approximately* 67 hundredths.

In line 2, we again see an error where *approximation by rounding* is treated as equality. In truth, $\frac{63}{250} = 0.252 = 25.2\%$. It is a terminating decimal, unlike two-thirds, but it is no more correct to round the number and state equality now than it was before. If we wish to express $\frac{63}{250}$ to the nearest whole percent, we must write $\frac{63}{250} \approx 25\%$.

In the last case the same mistake has been made. Pi is an irrational number—its digits never terminate nor repeat. Therefore it is incorrect to assign equality to a value of pi to be used in calculations. When finding areas of circles, for example, students are often told to use $\pi = 3.14$ or $\pi = \frac{22}{7}$. These contain incorrect notation and should be $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$.

The other major incorrect use of equals is in mathematical "run-on" sentences. For instance, if a student was told to find four more than two times three, a common result is $2 \times 3 = 6 + 4 = 10$. We understand what the student was thinking, but the written expression is wrong. All "sides" of the equation must be equal. Looking at the first two "sides" of the equation, we see that 2×3 is *not* equal to $6 + 4$. The student should be writing $2 \times 3 + 4 = 6 + 4 = 10$, which is a correct sentence.

It may be that teacher and student understand what the other means when an equation/approximation is written incorrectly, but such errors should be avoided to prevent misunderstanding.