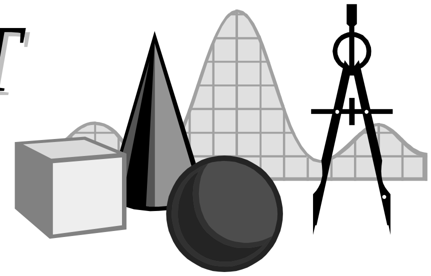
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
September 25, 2000 — High School Edition

Greetings! An enthusiastic welcome and welcome back to new and returning teachers from the Math Audit Team. With this issue, we embark on our second year of producing *Take It to the MAT*, a newsletter provided as a resource for teachers of all levels.

The Pythagorean Theorem is a key topic in the secondary school curriculum. As students get a handle on $a^2 + b^2 = c^2$ and learn to apply it, we hope that they memorize some of the nice sets of whole numbers that balance the equation. The ancient Eqyptians, Greeks, Babylonians, and Hindus all knew the magic of the 3-4-5 right triangle, and so should our students. There are more fascinating *Pythagorean triples* than the simple 3-4-5, and in this edition we will explore how to find them.

Pythagorean triples are sets of integers that satisfy the Pythagorean Theorem. Let's look at the first few triples that we know: 3-4-5, 5-12-13, 7-24-25. Are any patterns apparent? Several jump right out: a seems to be consecutive odd integers, b seems to be multiples of 4, and c is the whole number following b.

If we let n be some integer greater than or equal to 1, we could let $a = 2n + 1$ and would generate 3, 5, 7, 9, etc.

But what of b? The sequence 4, 12, 24 is clearly multiples of 4, namely $4 \cdot 1$, $4 \cdot 3$, and $4 \cdot 6$. Wow! Do you see it? The multipliers of 4 are 1, 3, 6—triangular numbers. The triangular numbers 1, 3, 6, 10, etc. are the result of a sequence which has a quadratic form. Specifically, if n is an integer greater than or equal to 1, the triangular numbers are $\frac{n(n+1)}{2}$. So $b = 4\frac{n(n+1)}{2}$.

Lastly, since $c = b + 1$, $c = 4\frac{n(n+1)}{2} + 1$. Cleaning up the equations a bit, we finish with $a = 2n+1$, $b = 2n^2 + 2n$, and $c = 2n^2 + 2n + 1$.

A quick verification of our work is shown below with a table of the first few triples.

n	**a**	**b**	**c**	**a^2**	**b^2**	**c^2**
1	3	4	5	9	16	25
2	5	12	13	25	144	169
3	7	24	25	49	576	625
4	9	40	41	81	1600	1681
5	11	60	61	121	3600	3721
6	13	84	85	169	7056	7225

$$
\begin{aligned}
&a^2 + b^2 \\
&= (2n+1)^2 + (2n^2+2n)^2 \\
&= (4n^2+4n+1) + (4n^4+8n^3+4n^2) \\
&= 4n^4 + 8n^3 + 8n^2 + 4n + 1 \\
&= (2n^2+2n+1)^2 \\
&= c^2
\end{aligned}
$$