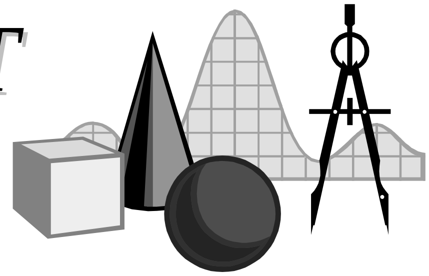
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
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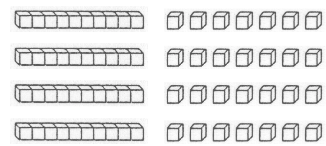
What do the distributive property, multiplication of two-digit numbers, and the area of rectangles have in common? Many things if one really thinks about it. In this issue of *Take It to the MAT*, we will explore the connections among the three topics mentioned.

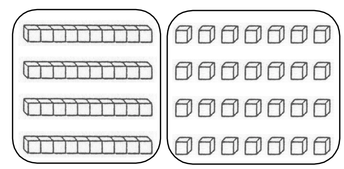
Let's start with the simple exercise 4×17. Should we immediately find the solution to this exercise by applying the multiplication algorithm as shown at right? Probably not. A modicum of number sense can easily lead us to the correct solution.

$$\begin{array}{r} {}^{2}\\ 17 \\ \times\ 4 \\ \hline 68 \end{array}$$

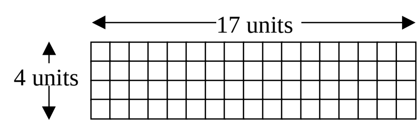
First, let us look at a model of 4×17 in base-ten blocks. When we see 4×17 we can think of four 17's, or perhaps 17 four times. Whatever the manner we choose to describe 4×17, the model shown represents the product.



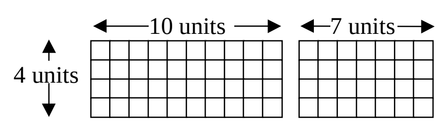
A look at the model reveals that there are four tens-rods and four groups of seven units. Four tens is 40, four groups of seven gives 28, so $4 \times 17 = 40 + 28 = 68$. Another way of examining this line of thinking is that $4 \times 17 = 4 \times (10 + 7)$. Notice that if 4 is multiplied by 10 and 7 separately, the products are 40 and 28 as we saw earlier. Thus, $4 \times (10 + 7) = (4 \times 10) + (4 \times 7)$. This is the *distributive property of multiplication*—more generally, $a \times (b + c) = (a \times b) + (a \times c)$.



Another way to view 4×17 is that of a rectangular array. If we look at the base-ten block model above, we can see that it is 4 units wide by 17 units long. An alternate view of the model is that of a 4×17 grid as shown at right.



What is the area of a rectangle that is 4 units wide by 17 units long? Count the square units! There are 68 square units in the rectangle. But if counting all 68 squares is too tedious,—it is—or if the product of 4×17 does not immediately pop into one's head,—it shouldn't—we can restructure this model as we did with the base-ten blocks. We'll break it into two smaller rectangles, one 4×10 and the other 4×7. We can easily multiply those two expressions mentally. The smaller rectangles have areas of 40 and 28 square units, respectively. The total area is 68 square units.



A last way to see this exercise that connects to what we have just explored is through an algorithm, although not the typical one we teach students. (Before we go on it must be said that 4×17 is *not* an exercise requiring an algorithm—it should be done mentally. The linkage is made here in preparation for products that will require an algorithm.) In this algorithm we will multiply from "bottom up" then from "right to left", but only focusing on one digit in each factor at a time. First ones by ones (4×7), then ones by tens (4×10). This relates very well to our models using cubes and area, as well as the distributive property.

$$\begin{array}{rl} 17 & \\ \times\ 4 & \\ \hline 28 & = 4\times7 \\ 40 & = 4\times10 \\ \hline 68 & \end{array}$$

Connections among these three topics must be made in order for students to link visual representations to properties and algorithms.