TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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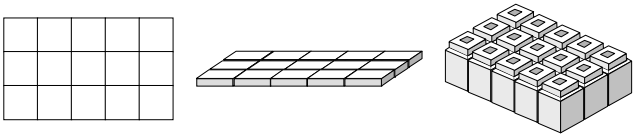
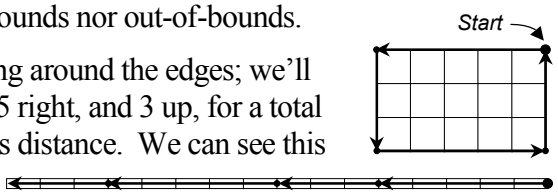
When teaching students abstract ideas it is important to model them concretely as much as possible. Two concepts that we typically model with manipulatives are area and perimeter. Unfortunately, students can develop misconceptions if we are not careful about the models we use.



To model area we often use graph paper, square pattern blocks, or cubes as shown at right. Students learn to count the blocks, count the squares, or count the cubes. In the case of graph paper, the model is sound. *Area* is indeed the number of one-unit squares inside a figure. With pattern blocks, we're still on fairly firm ground due to the thinness of the tiles. With cubes we should be a bit more concerned.

Since area is a two-dimensional measurement, modeling with more obvious three-dimensional objects such as cubes can leave students with misconceptions. "Counting cubes" to find area is not entirely accurate. They should in fact be counting the tops of the cubes or the "footprint" of the cubes. Essentially, students must be looking straight down on the cubes. Even pattern blocks and tiles are 3-D objects and students must understand what they are counting. Therefore, we must be very careful if we choose to use cubes to develop the concept of area and in fact may do well to avoid cubes altogether when teaching area.

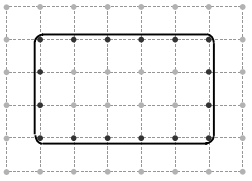
The concept incorrectly modeled most often is that of *perimeter*. Perimeter literally means "distance around." Too often we incorrectly describe it as the "outside" of the figure. Distance is a linear—one-dimensional—measurement. Therefore counting cubes, tiles, or squares makes no sense. We should imagine perimeter as how far one would "walk" around a figure *on its edges*, like walking around the boundary lines of a basketball court, but not walking in-bounds nor out-of-bounds.



Examine the rectangle at right and picture yourself walking around the edges; we'll start at the upper right. You walked 5 units left, 3 down, 5 right, and 3 up, for a total of 16 units. The 16 units is the perimeter, and perimeter is distance. We can see this by "unfolding" our path and seeing it as a straight line.

What makes no sense when discussing perimeter is "counting squares" around the outside. We count squares when we want area, not perimeter. Notice what happens when we count squares (or tiles, or cubes) to find perimeter—the result is 12, which is incorrect.

We must also be careful how we develop perimeter with the use of Geoboards. Students need to count the unit lengths *between* the posts and not the posts along the rectangle. While both methods provide the same answer, only one is clearly connected to the number of units around a figure.



Modeling measurements of 2-D geometric figures—perimeter and area—with manipulatives is a good way to develop concepts before proceeding to mathematical formulas. We must always be sure that our models make sense and do not leave students with misconceptions.