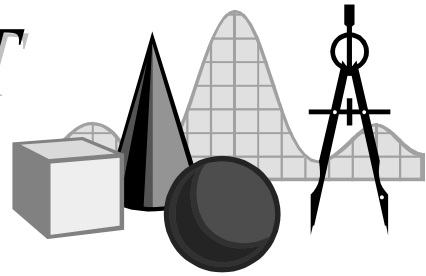


TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

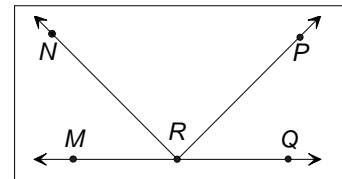
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One of the branches of mathematics where notation is used extensively is in geometry. The sheer quantity of the various symbols used is sometimes overwhelming, especially to students. In this issue of *Take It to the MAT*, we will examine some common geometric notation and how it is sometimes used incorrectly or misrepresented.

Examine the figure at right and describe what the following mean: \overleftrightarrow{MQ} , \overrightarrow{MQ} , \overline{MQ} , and MQ .

The first, \overleftrightarrow{MQ} (read, “line M-Q”), is the *line* passing through the points M and Q . The line continues infinitely in both directions, and can also be written \overleftrightarrow{QM} .



The second, \overrightarrow{MQ} (“ray M-Q”), is the *ray* whose *initial point/endpoint* is at M and continues on infinitely, through Q , in only one direction. The “arrow” is always written pointing to the right, with the first point listed being the endpoint and the second point listed as any other point the ray passes through. Thus, \overrightarrow{MQ} and \overrightarrow{MR} are the same ray, but \overrightarrow{QM} is not.

The third, \overline{MQ} (“segment M-Q”), is that part of the line \overleftrightarrow{MQ} consisting of the points between M and Q , inclusive. Like the notation for line, \overline{MQ} could also be written as \overline{QM} .

The last, MQ (“M-Q”), represents the *distance* between the points M and Q , or the *length* of segment \overline{MQ} . Sometimes, the length of segment \overline{MQ} is represented by $m\overline{MQ}$ (“measure of segment M-Q”).

Consequently, MQ is a number; \overline{MQ} is a set of points.

Now that we’ve dealt with lines, rays, and segments, let’s examine angles. Look at the figure again and describe the angle formed by rays \overrightarrow{RM} and \overrightarrow{RN} . The correct notation is $\angle MRN$ (“angle M-R-N”) or $\angle NRM$ —either is correct, however $\angle R$ is not correct. But, where many of us make a mistake is in describing the measure of the angle. Let’s say that $\angle MRN$ measures 45° . How does one write that? Often, we *incorrectly* write $\angle MRN = 45^\circ$, whereas the correct notation is $m\angle MRN = 45^\circ$. That is, “the *measure* of $\angle MRN$ is 45° .”

Many times, we also mix up the difference between *equality* and *congruence*. Consider the fact that the point R is the midpoint of \overline{MQ} . Does that mean $\overline{MR} = \overline{RQ}$? Certainly not! *Equality* relates to the *measures* of geometric figures; *congruence* relates to the figures themselves. (Two *figures* are congruent if they can be made to coincide by performing translations, rotations, or reflections of them in space.) Thus, we can say $MR = RQ$ (“M-R equals R-Q”), or we can say $\overline{MR} \cong \overline{RQ}$ (“segment M-R is congruent to segment R-Q”).

As for angles, let’s say $\angle PRQ$ measures 45° , just like $\angle MRN$. So, we can say $\angle MRN \cong \angle PRQ$ (“angle M-R-N is congruent to angle P-R-Q”), or that $m\angle MRN = m\angle PRQ$ (“the measure of angle M-R-N is equal to the measure of angle P-R-Q”), but we may not say $\angle MRN = \angle PRQ$. Once again, figures are congruent; measures are equal.

Often we get sloppy and use notation interchangeably, as if any one symbol were the same as another. Because of the symbolic nature of mathematical language we must avoid this. It would be like saying any too similar-looking words in the English language have the same meaning.