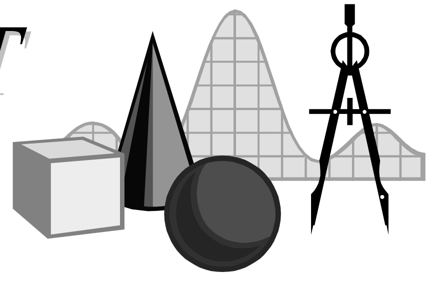
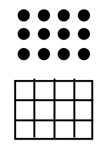
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

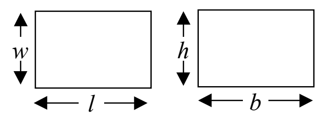


Math Audit Team
Regional Professional Development Program
January 29, 2001 — High School Edition

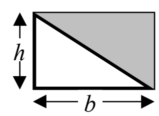
Calculating areas of triangles and quadrilaterals is an important skill and is featured on the Nevada High School Proficiency Exam. Students frequently don't understand formulas because they don't see connections between them or to their relevant figures. In this issue of *Take It to the MAT* we will look at *developing* area formulas in a little different manner than traditional textbooks.



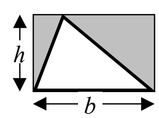
We begin with the rectangle. Students already know something of the area of rectangles from using arrays or lattices to learn multiplication facts. The connection is strong between a 3×4 matrix having 12 elements and a 3 inch $\times$ 4 inch rectangle having an area of 12 square inches. The area of the rectangle is essentially counting the 1×1 squares of the 3×4 grid. (Students should see this link prior to being given the formula.)

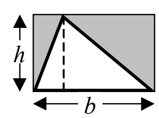


Now, on to the shortcut formula. It does not take long to figure out that the area of the rectangle is the product of its length and width; $A = lw$. Let us consider the dimensions in a different way. We will refer to the horizontal dimension as the *base* and the vertical dimension as *height.* Thus, $A = bh$.

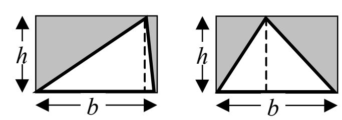


We have established that the area of a rectangle is *base* $\times$ *height*. Draw a diagonal in the rectangle to create two triangles. What is the area of each of the triangles? Since the two triangles are congruent, each must have half of the original area. Thus, the area of the triangle is *half* of *base* $\times$ *height*, that is $A = \frac{1}{2}bh$.

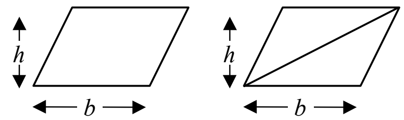




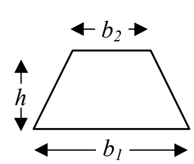
The formula $A = \frac{1}{2}bh$ at least applies to the right triangle, but what of non-right triangles? Look again at that rectangle, its diagonal, and one of the right triangles formed by them. Now, grab the upper left vertex of the unshaded triangle and "slide" it across the top side of the rectangle. Can you see that the area of the unshaded triangle and the shaded area outside the triangle are equal? If not, draw an altitude from the moved vertex. You should now see two sets of congruent right triangles.



It does not matter where the vertex is "slid" along the side of the rectangle; the resulting triangle has the same base length and height. The area of any triangle is therefore $A = \frac{1}{2}bh$.



The triangle formula is now developed, so let's move on to the parallelogram. What is the area of a parallelogram with a base length of b and a height of *h*? Draw that diagonal again. The diagonal cuts the parallelogram into two triangles, both having base *b* and height *h*. So the area of the parallelogram is *twice* the area of either triangle, $\frac{1}{2}bh$, or $A = 2 \times \frac{1}{2}bh = bh$.



The last figure that we address is the trapezoid. The formula for a trapezoid's area is $A = \frac{1}{2}(b_1 + b_2)h$. The derivation of that formula using the ideas developed above is left to the reader.