

Interesting Mathematics Patterns

Strategies for learning math facts are provided on www.rpd.net. Some students learn to see patterns and math strategies as they learn the basic math facts. Everyone likes to “figure out the trick.” What is nice about mathematics is that these patterns are not really tricks, they have mathematical basis.

In addition to the strategies for learning basic math facts such as multiplication or division by 10, there are many interesting math patterns. Look at the following samples. Do you see the patterns?

Squaring a number ending in 5

$$\begin{aligned} 15^2 &= 225 \\ 25^2 &= 625 \\ 35^2 &= 1225 \\ 45^2 &= \end{aligned}$$

Hint: The square will always end in 25. The numbers in front of the 25 will be the product of the (first digit) times the (first digit+1). So $45^2 = (4)(5)$ or 20 then append 25, making 2025. Does it work for three digit numbers such as 115^2 ?

Multiplying two digit numbers with the same first number and with the ones digits adding to 10

$$\begin{aligned} 12 \cdot 18 &= 216 \\ 21 \cdot 29 &= 609 \\ 34 \cdot 36 &= 1224 \\ 47 \cdot 43 &= 2021 \\ 97 \cdot 93 &= \end{aligned}$$

Hint: This is very similar to squaring numbers ending in 5. The product will always end in the two digit product of the units digits of the factors. The numbers in front of that product will be the product of the (first digit) times the (first digit+1).

Does it work for three digit numbers? Try 113 times 117.

A Miscellaneous Pattern

$$\begin{aligned} 1 \cdot 8 + 1 &= 9 \\ 12 \cdot 8 + 2 &= 98 \\ 123 \cdot 8 + 3 &= 987 \\ 1234 \cdot 8 + 4 &= 9876 \\ 12345 \cdot 8 + 5 &= \\ 123456 \cdot 8 + 6 &= \\ &\dots \end{aligned}$$

Patterns with repeating decimals

$$\begin{aligned} \frac{1}{9} &= .111\dots \text{ or } \overline{.1} \\ \frac{2}{9} &= \\ \frac{3}{9} &= \end{aligned}$$

Have you noticed that certain fractions produce repeating decimals?

Repeating decimals can be written as fractions with special denominators.

Remember that the bar above the repeating portion is called a vinculum.

Look at these extended patterns. (Note: Fractions are not simplified.)

$$\begin{aligned} \overline{.12} &= \frac{12}{99} & \overline{.123} &= \frac{123}{999} \\ \overline{.13} &= \frac{13}{99} & \overline{.154} &= \frac{154}{999} \\ \overline{.27} &= \frac{27}{99} & \overline{.276} &= \frac{276}{999} \end{aligned}$$

$$\begin{aligned} \overline{.015} &= \frac{15}{990} & \overline{.22} &= \frac{20}{90} & \overline{.215} &= \frac{213}{990} & \overline{.235} &= \frac{212}{900} \\ \overline{.024} &= \frac{24}{990} & \overline{.35} &= \frac{32}{90} & \overline{.324} &= \frac{321}{990} & \overline{.3759} &= \frac{3722}{9900} \\ \overline{.0276} &= \frac{276}{9990} & \overline{.427} &= \frac{423}{990} & \overline{.4276} &= \frac{4272}{9990} & \overline{.4276} &= \frac{4234}{9900} \end{aligned}$$

Hint: Did you figure it out? Try more examples.

Multiplication by 11

Hint: Multiplication by 11 is easy! To multiply by a 2-digit number add the two digits and place the sum in between the first and second digit!

If the sum you need to place in between is more than 9, you need to carry the 1!

What about a 3-digit number? Can you figure out what's going on here?

Do examples! What you notice is that multiplication by 11 can be done quickly with numbers of any length by starting with the first and last digits (they remain the same, unless there is a carry) and then inserting the sums of adjacent pairs of digits sequentially in between. For example, 253×11 begins with a 2 (like 253 does), then the next digit is $2+5=7$, the next digit is $5+3=8$, and the last digit is 3 (like 253 has). So the product is 2783. Remember to carry if necessary. So 267×11 starts with a 2 (like 267 does), the next digit is $2+6=8$, the next digit is $6+7=13$ (Oops! Carry the 1 back to the previous digit, leaving a 3 in this place), then the last digit is 7 (just like 267 has). Thus the product is 2937. (After you get used to it, you may want to work from right to left - it makes carrying easier.)



Math Resources

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